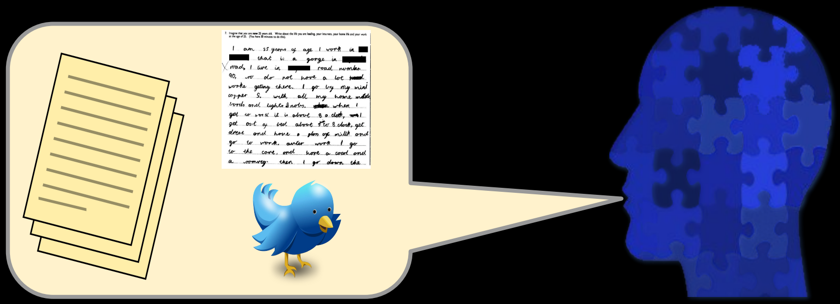
Language Modeling

CSE354 - Spring 2021

Task

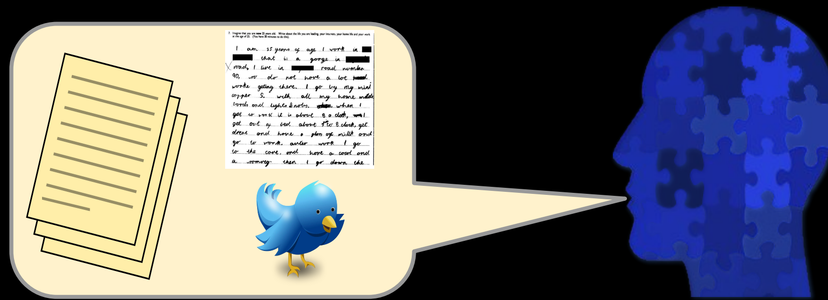


- Language Modeling (i.e. auto-complete)

how? →

- **Probabilistic Modeling**
  - **Probability Theory**
  - **Logistic Regression**
  - **Sequence Modeling**

Task



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how? →

- Probabilistic Modeling
  - Probability Theory
  - Logistic Regression
  - Sequence Modeling
- **Eventually: Deep Learning**
  - **Recurrent Neural Nets**
  - **Transformer Networks**

Language Modeling

-- assigning a probability to sequences of words.

Version 1: Compute $P(w1, w2, w3, w4, w5) = P(W)$

:probability of a sequence of words

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Version 2: Compute $P(w5 | w1, w2, w3, w4)$

$= P(w_n | w_1, w_2, ..., w_{n-1})$

:probability of a next word given history

Language Modeling

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P(He ate the cake with the fork) = ?

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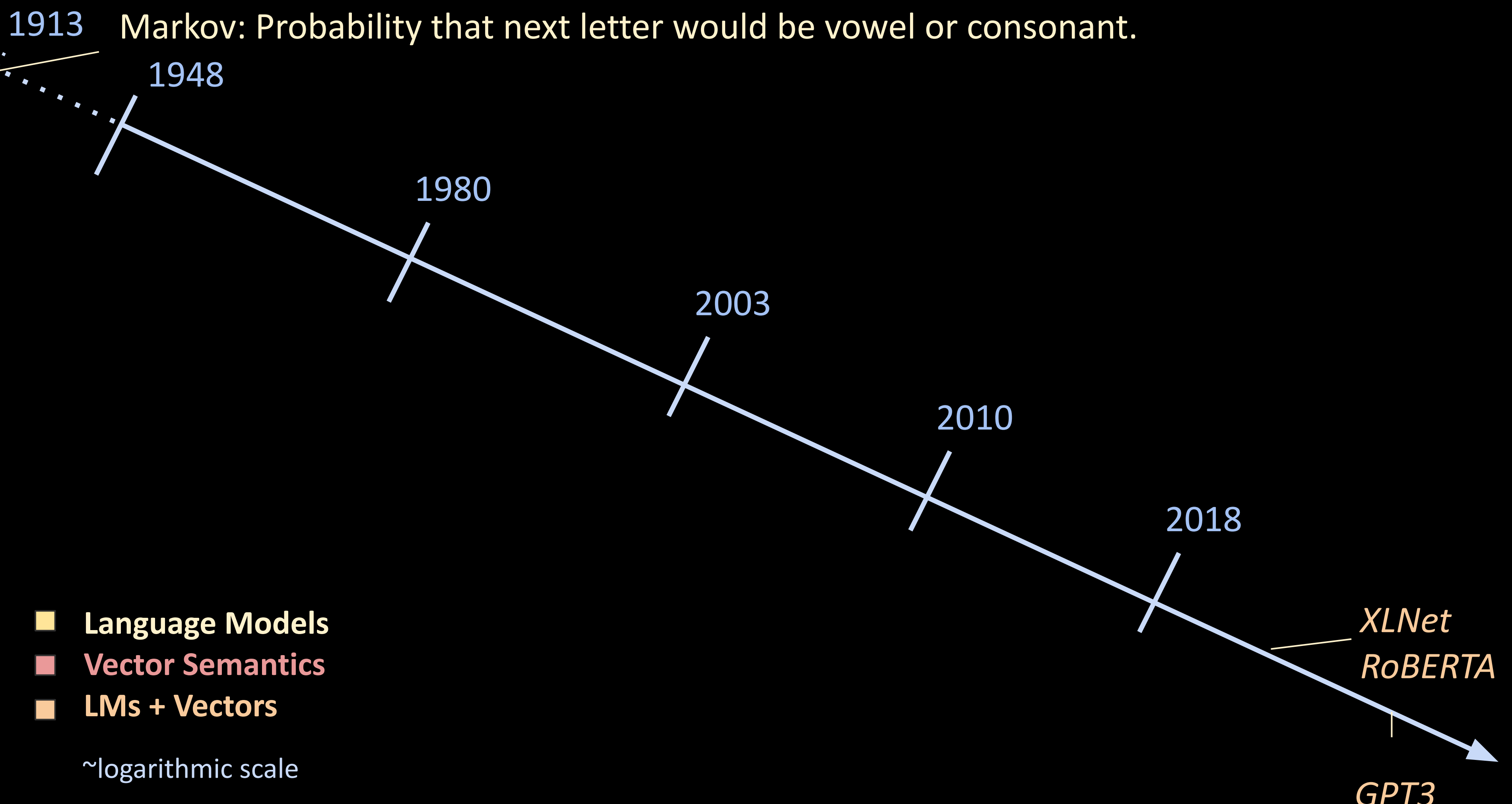
Language Modeling

Applications:

- Auto-complete: What word is next?
- Machine Translation: Which translation is most likely?
- Spell Correction: Which word is most likely given error?
- Speech Recognition: What did they just say?
  “eyes aw of an”
  (example from Jurafsky, 2017; ..did you say "giraffe ski 2,017"?)

Timeline: *Language Modeling* and *Vector Semantics*



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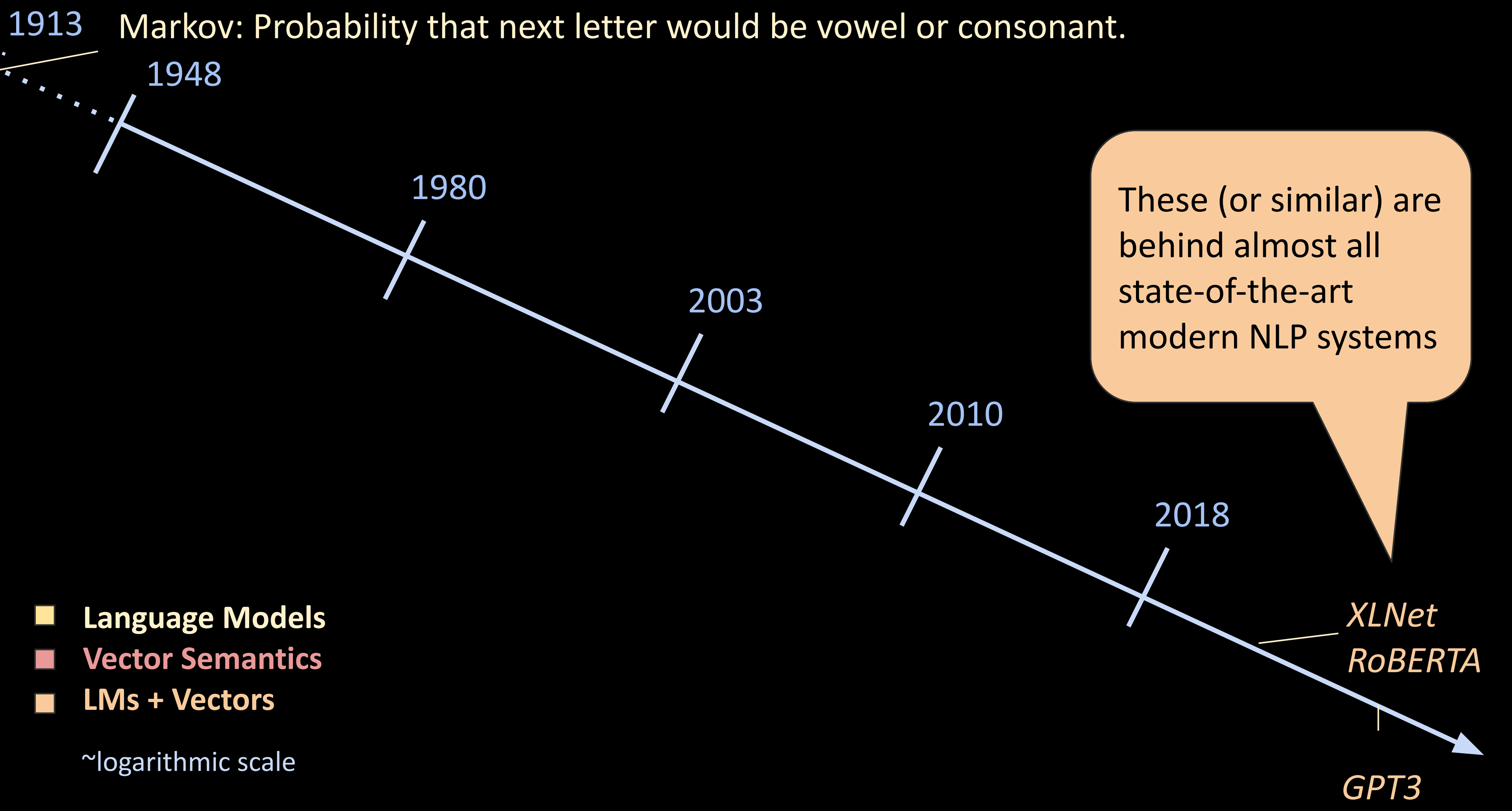

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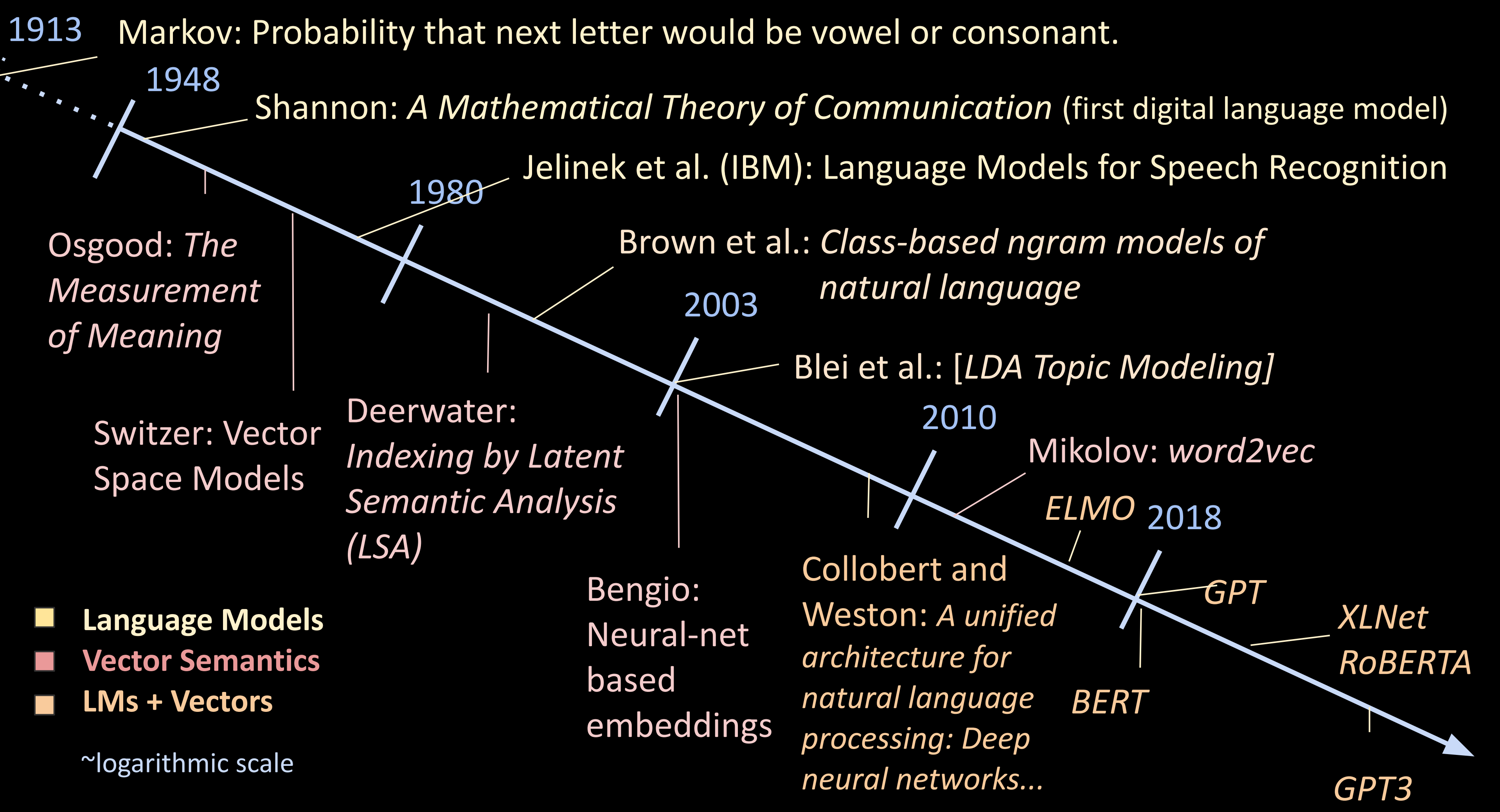

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Simple Solution

Version 1: Compute $P(w1, w2, w3, w4, w5) = P(W)$

:probability of a sequence of words

$P(\text{He ate the cake with the fork}) =$

$$\frac{count(\text{He ate the cake with the fork})}{count(*\ *\ *\ *\ *\ *\ *)}$$

Simple Solution: The Maximum Likelihood Estimate

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$P(\text{He ate the cake with the fork}) =$

$$\frac{count(\text{He ate the cake with the fork})}{count(* \quad * \quad * \quad * \quad * \quad * \quad *)}$$

total number of observed *7grams*

Simple Solution: The Maximum Likelihood Estimate

$$P(\textit{He ate the cake with the fork}) = \frac{count(\textit{He ate the cake with the fork})}{count(* \quad * \quad * \quad * \quad * \quad * \quad *)}$$

$$P(\textit{fork} \mid \textit{He ate the cake with the}) = \frac{count(\textit{He ate the cake with the fork})}{count(\textit{He ate the cake with the } *)}$$

Simple Solution: The Maximum Likelihood Estimate

Problem: even the Web isn't large enough to enable good estimates of most phrases.

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Example from (Jurafsky, 2017)

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The Chain Rule:

$$P(X_1, X_2, \ldots, X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2)\ldots P(X_n|X_1, \ldots, X_{n-1})$$

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The Chain Rule: $P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i|X_1, X_2, ..., X_i)$

$$P(X1, X2,..., Xn) = P(X1)P(X2|X1)P(X3|X1, X2)...P(Xn|X1, ..., Xn\text{-}1)$$

Markov Assumption: $$P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i | X_{i-k}, X_{i-(k-1)}, ..., X_i)$$

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Markov Assumption:

$P(X \ldots X_i) \ldots$ $\ldots 1), \ldots, X_i)$

P(Xn| X…

What about Logistic Regression? Y = next word
P(Y|X) = P(Xn | Xn-1, Xn-2, Xn-3, ...)

Not a terrible option, but Xn-1 through Xn-k would be modeled as independent dimensions. Let's revisit later.

The Chain… $\ldots \prod_{i=1}^{n} P(X_i|X_1, X_2, \ldots, X_i)$

P(X1, X2,..., Xn) = P…(X2|X1)P(X3|X1, X2)...P(Xn|X1, ..., Xn-1)

Markov Assumption: $$P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i | X_{i-k}, X_{i-(k-1)}, ..., X_i)$$

$P(Xn| X1..., Xn\text{-}1) \approx P(Xn| Xn\text{-}k, ..., Xn\text{-}1)$ *where* $k < n$

Unigram Model: k = 0; $$P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i)$$

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$$P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i | X_{i-k}, X_{i-(k-1)}, ..., X_i)$$

P(Xn| X1..., Xn-1) ≈ P(Xn| Xn-k, ..., Xn-1) where k < n

Bigram Model: k = 1;

$$P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i | X_{i-1})$$

Example generated sentence:

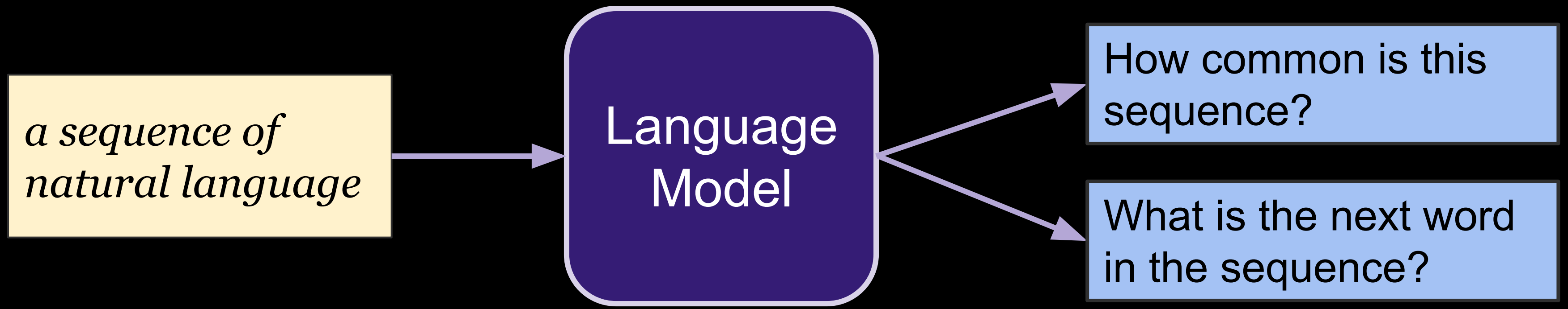
outside, new, car, parking, lot, of, the, agreement, reached

P(X1 = "outside", X2="new", X3 = "car", ...)
*≈ P(X1="outside") * P(X2="new"|X1 = "outside) * P(X3="car" | X2="new") * ...*

Example from (Jurafsky, 2017)

Language Modeling

Building a model (or system / API) that can answer the following:



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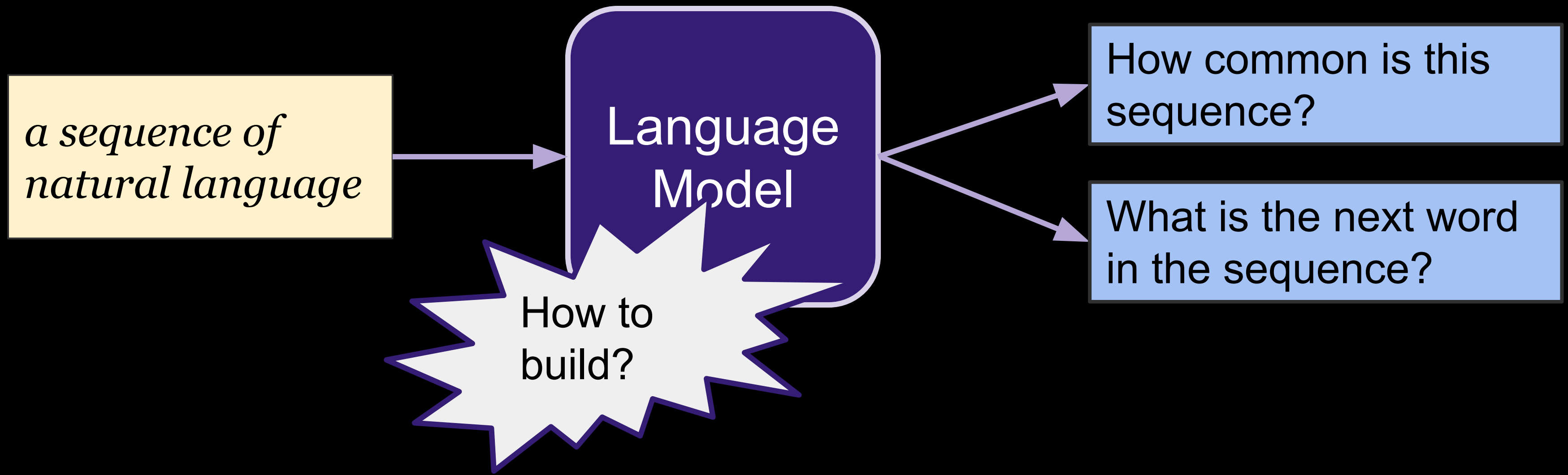

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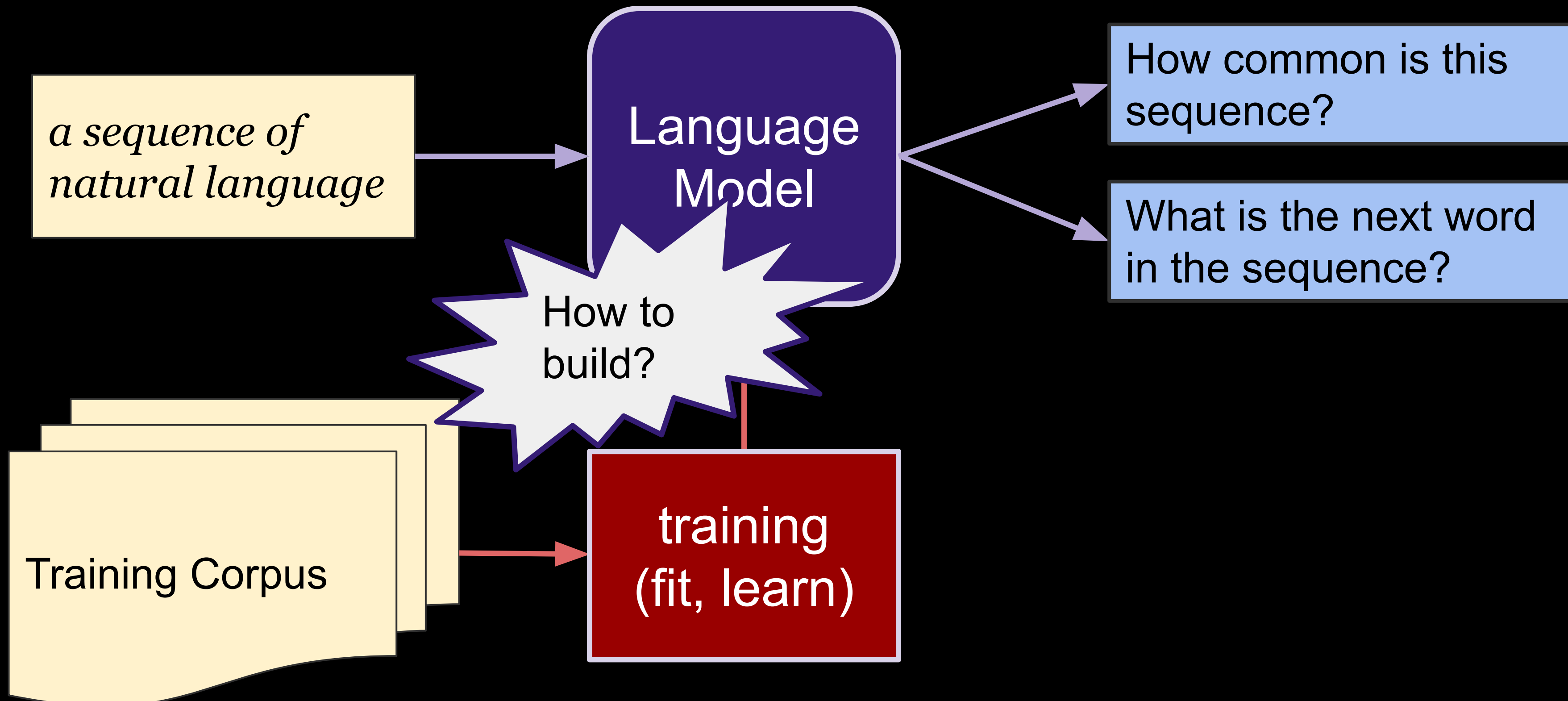


Language Mo

Building a model (

Food corpus from Jurafsky (2018). Samples:

can you tell me about any good cantonese restaurants close by

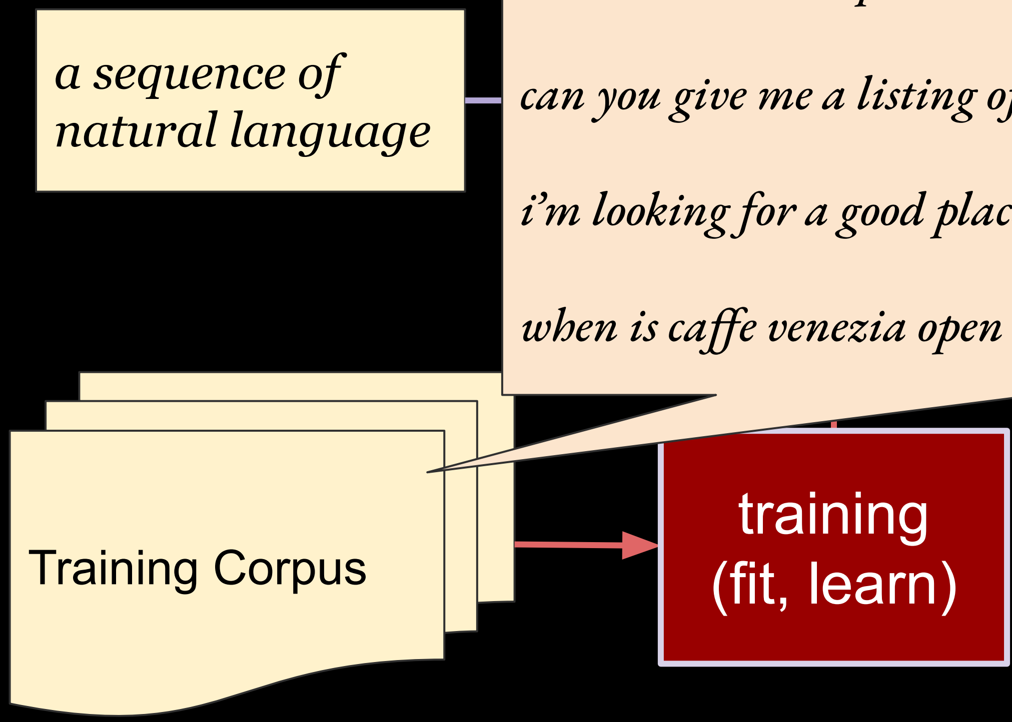
mid priced thai food is what i'm looking for

tell me about chez panisse

can you give me a listing of the kinds of food that are available

i'm looking for a good place to eat breakfast

when is caffe venezia open during the day

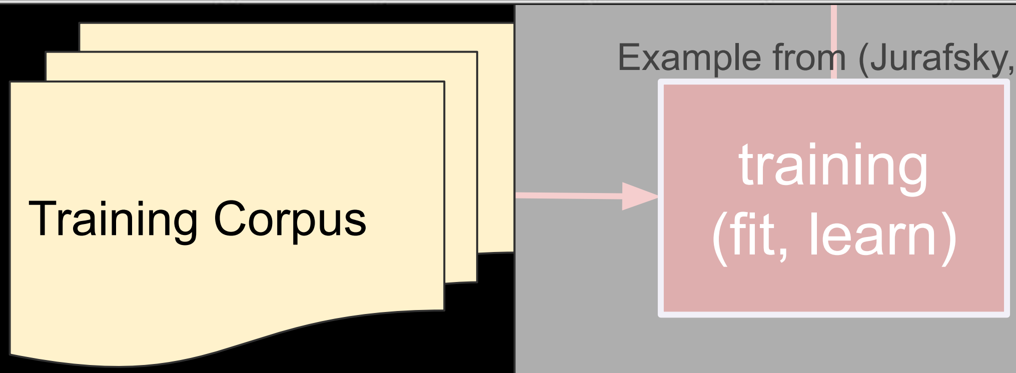


Bigram Counts

first word / second word

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Example from (Jurafsky, 2017)



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i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

Training Corpus

training
(fit, learn)

Bigram Counts

first word \ second word

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Bigram model: $P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i | X_{i-1})$

Need to estimate: *P(Xi | Xi-1)* = count(Xi-1 Xi) / count(Xi-1)

second word: xi

first word: xi-1

P(Xi | Xi-1)

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

i	want	to	eat	chinese	food	lunch	spend
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first word(Xi-1) \ second word (Xi)

P(Xi | Xi-1)

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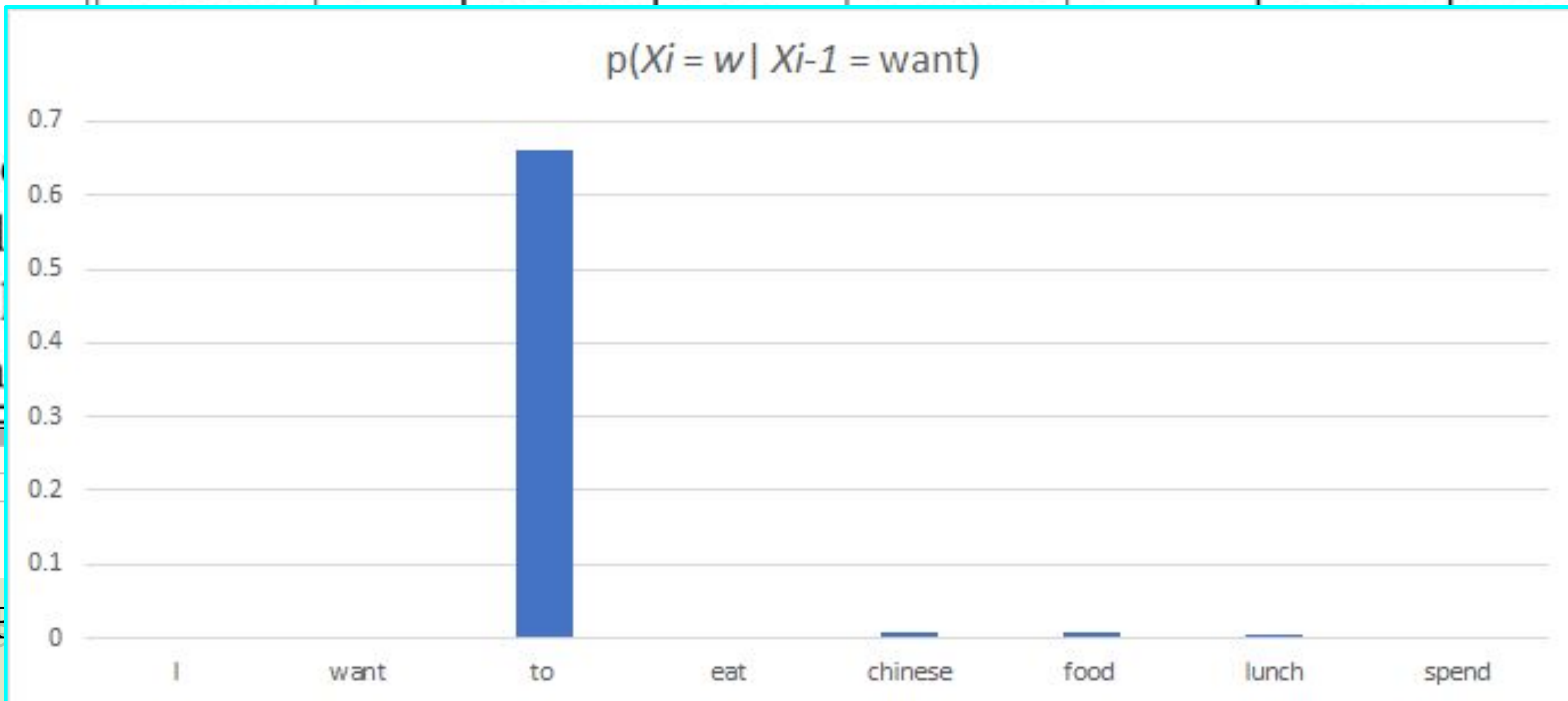

Need to estimate: $P(X_i \mid X_{i-1}) = \text{count}(X_{i-1}\, X_i) / \text{count}(X_{i-1})$

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want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
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								0
								0

0
0
0

spend
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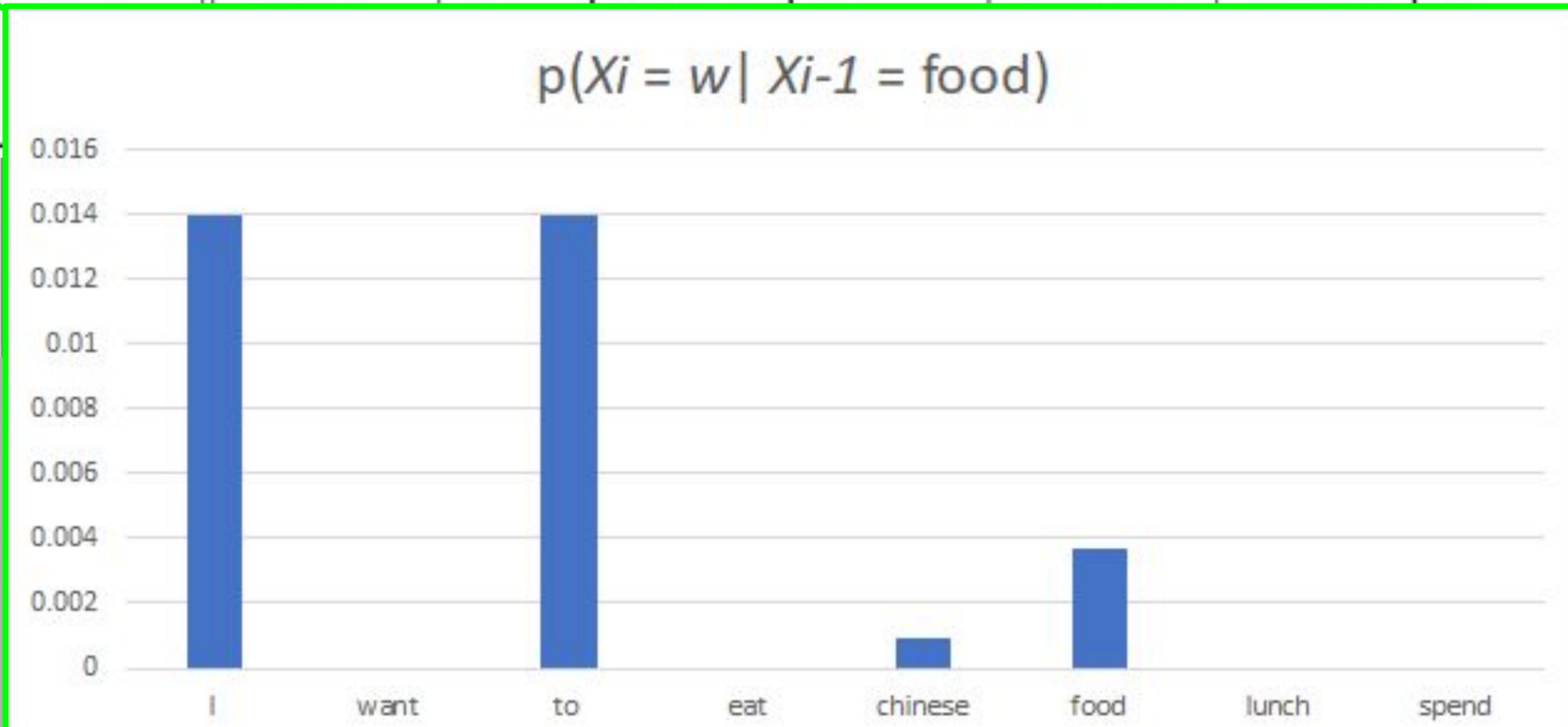

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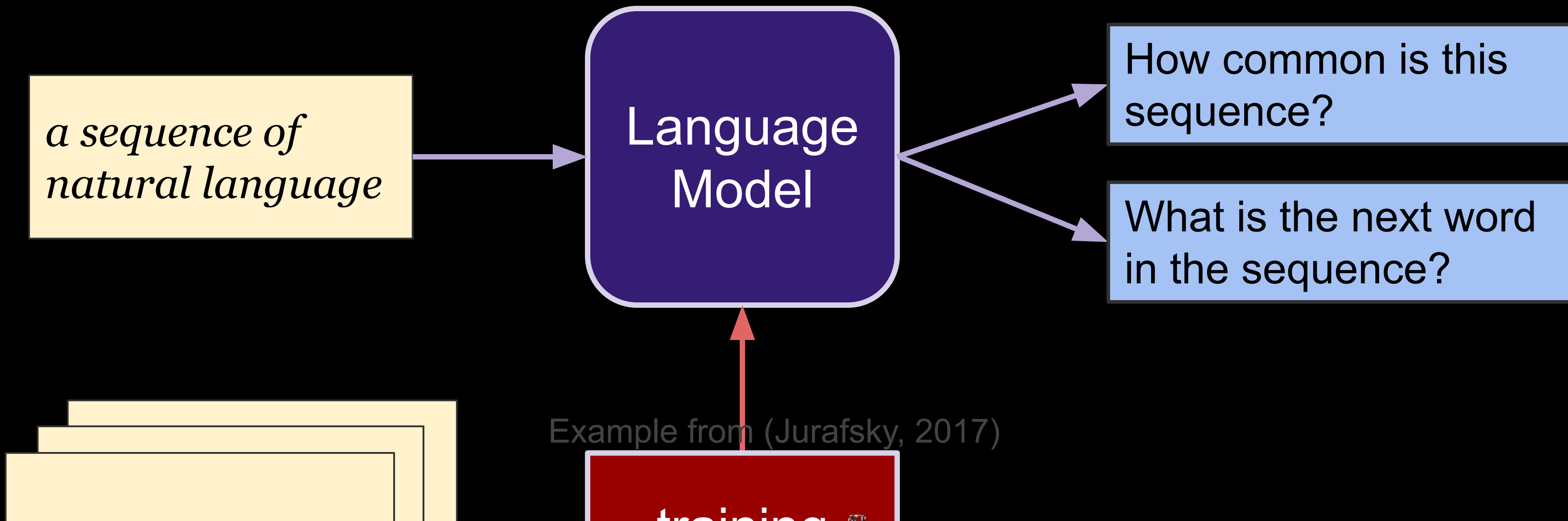

Example from (Jurafsky, 2017)

Bigram model: $P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | X_{i-1})$

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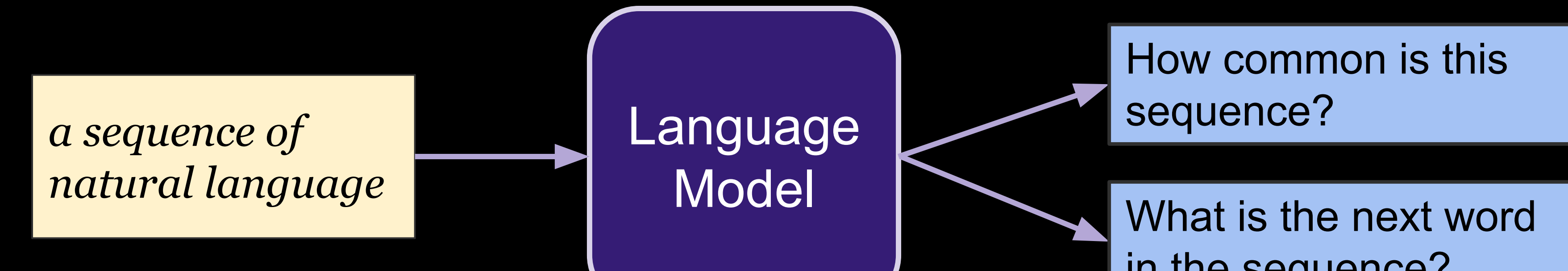

Trigram model: $$P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i | X_{i-2}, X_{i-1})$$

Need to estimate: *P(Xi | Xi-1, Xi-2)* = count(Xi-2 Xi-1 Xi) / count(Xi-2 Xi-1)

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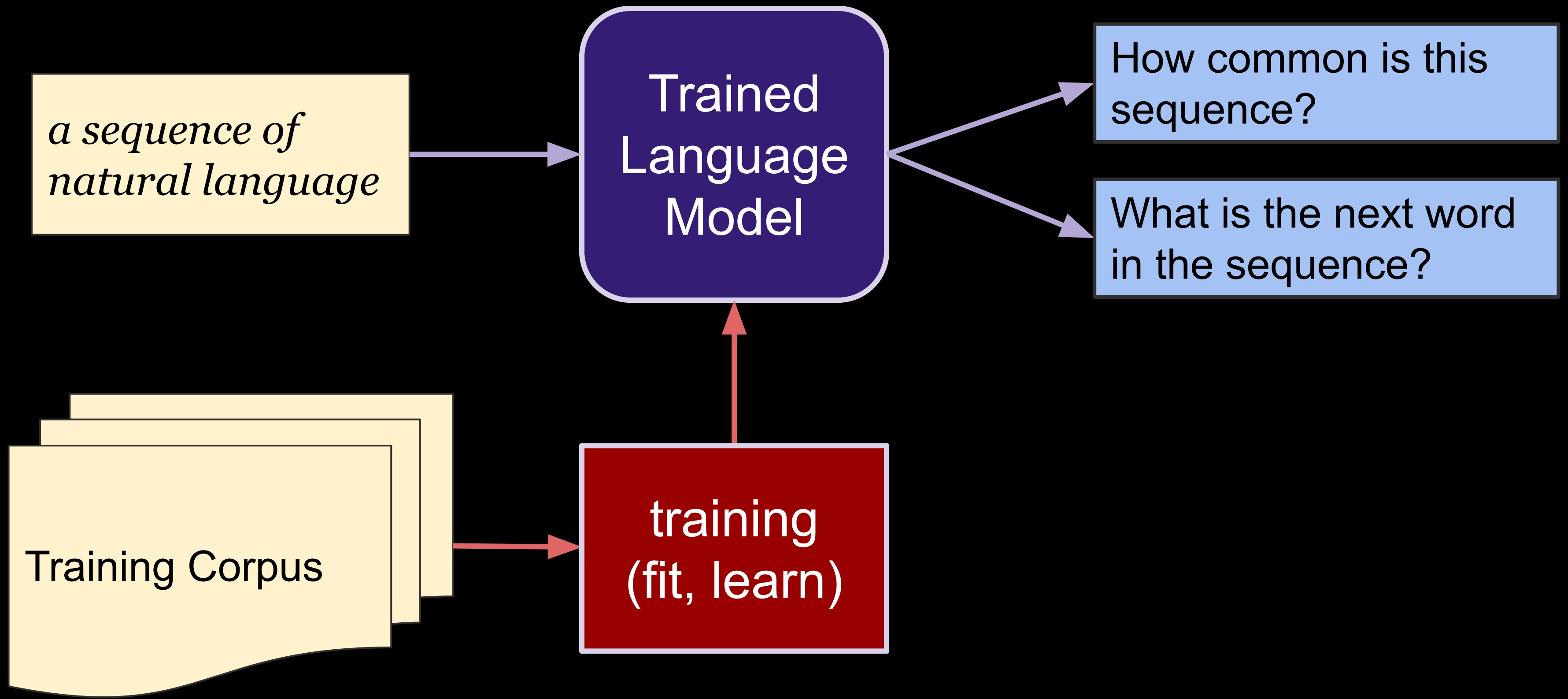

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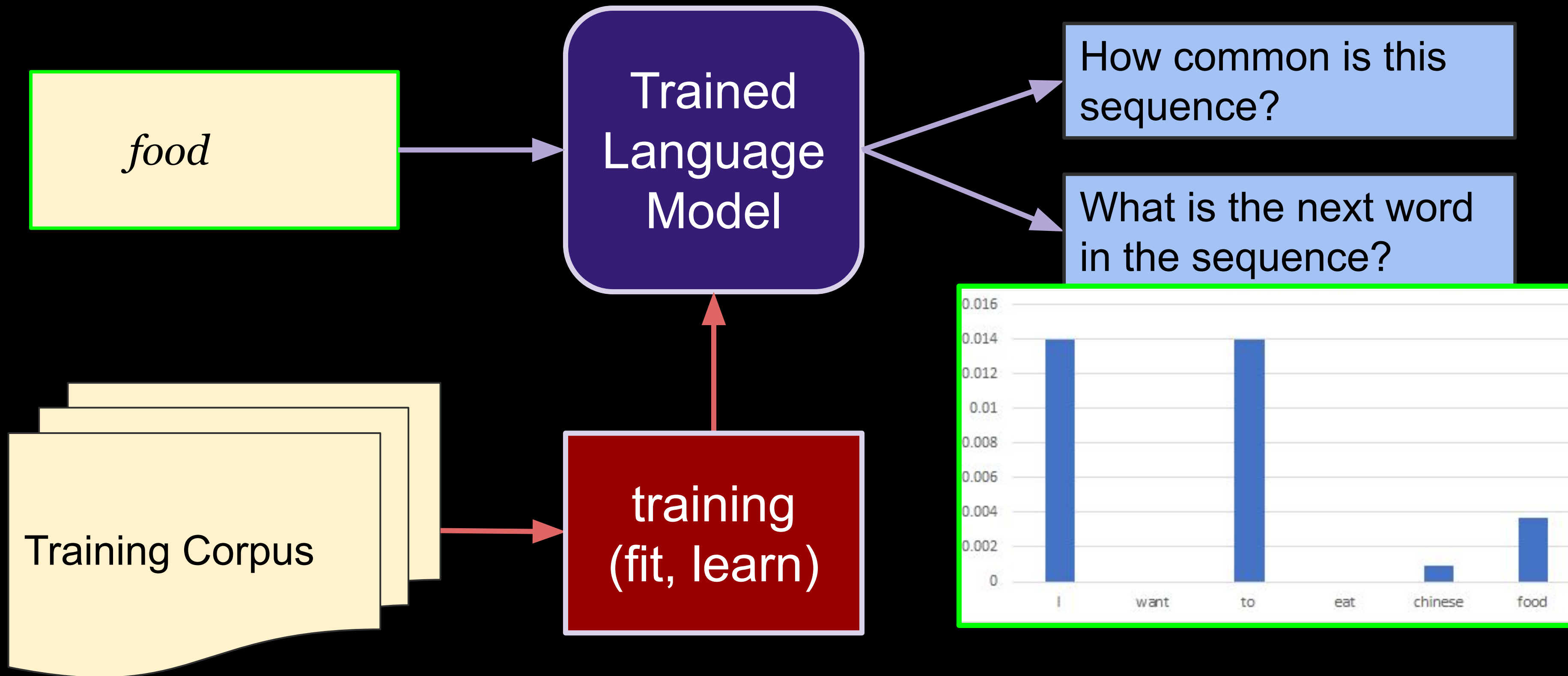

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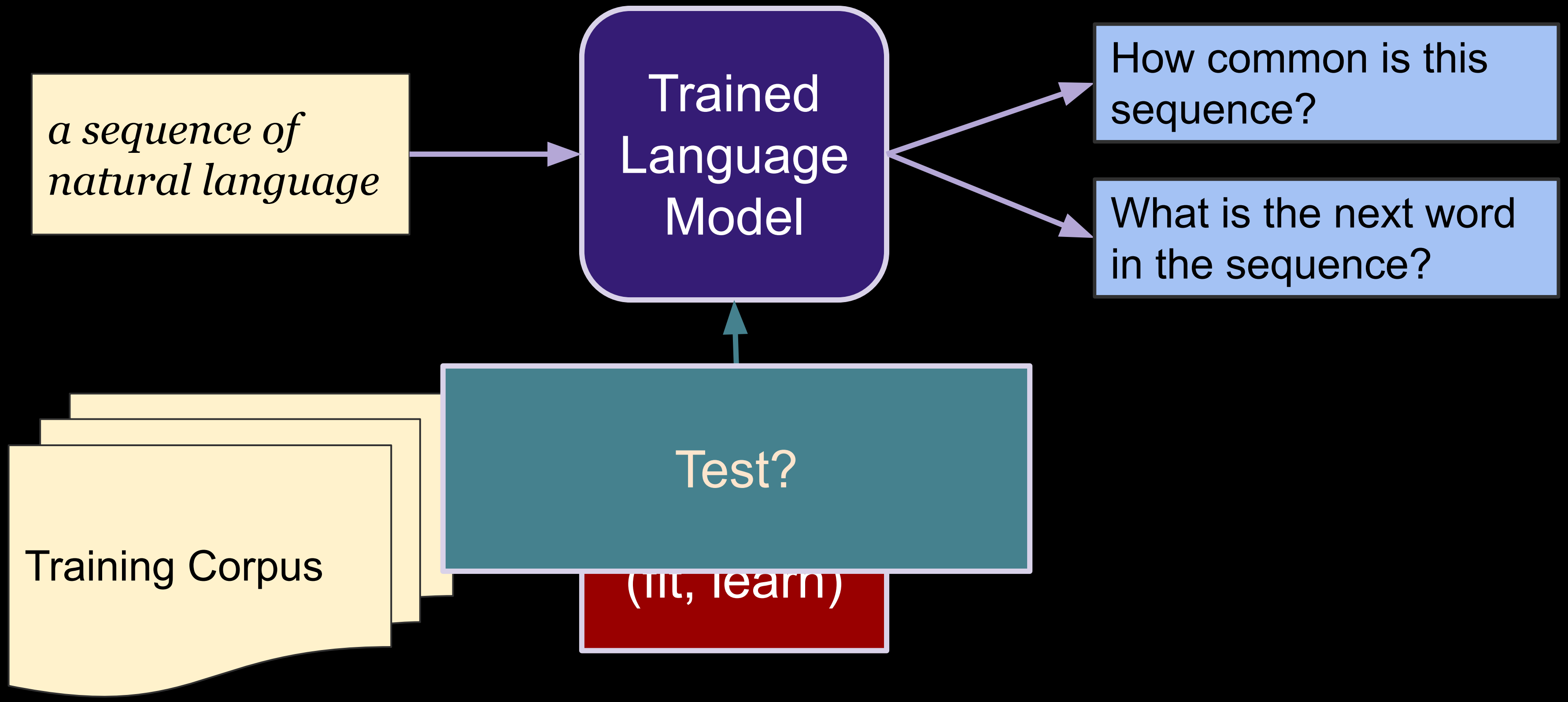

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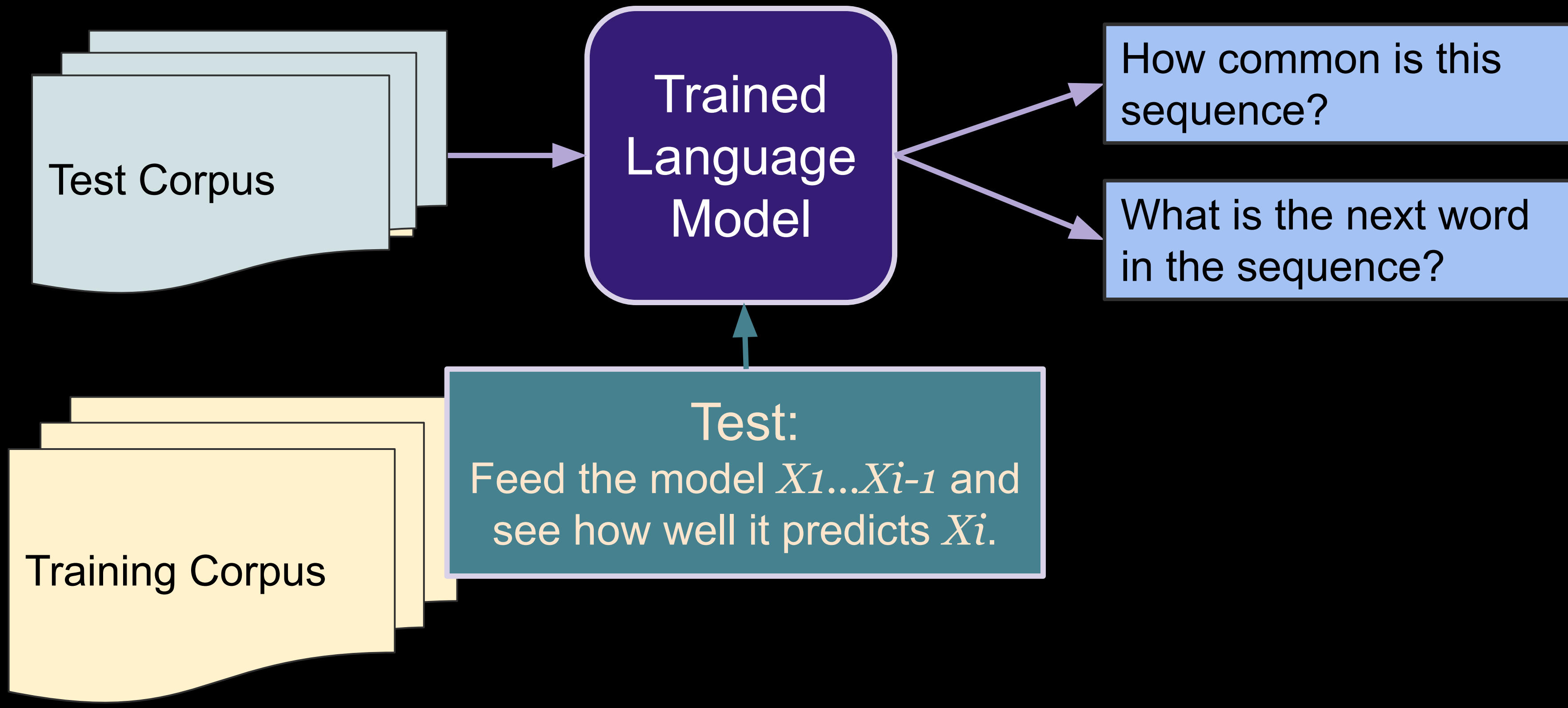

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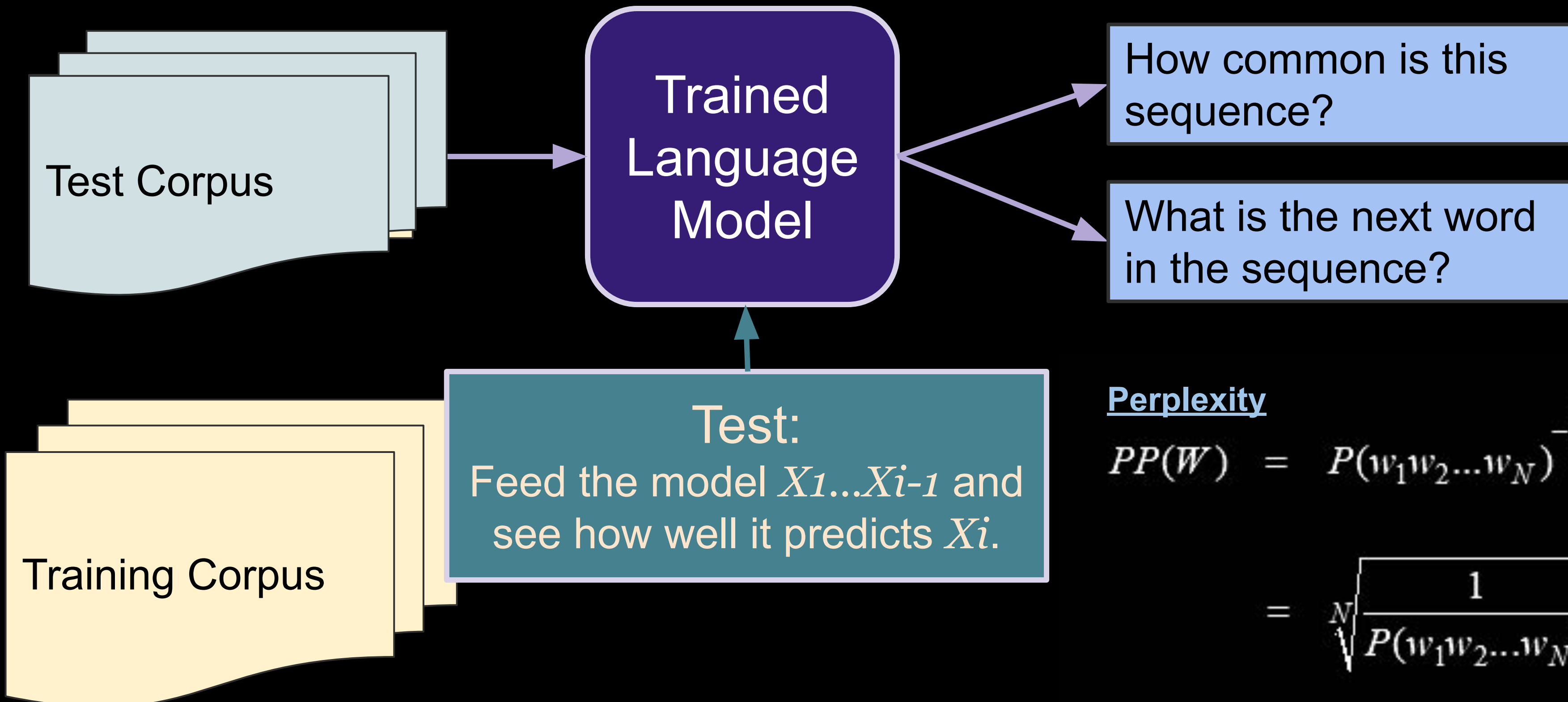
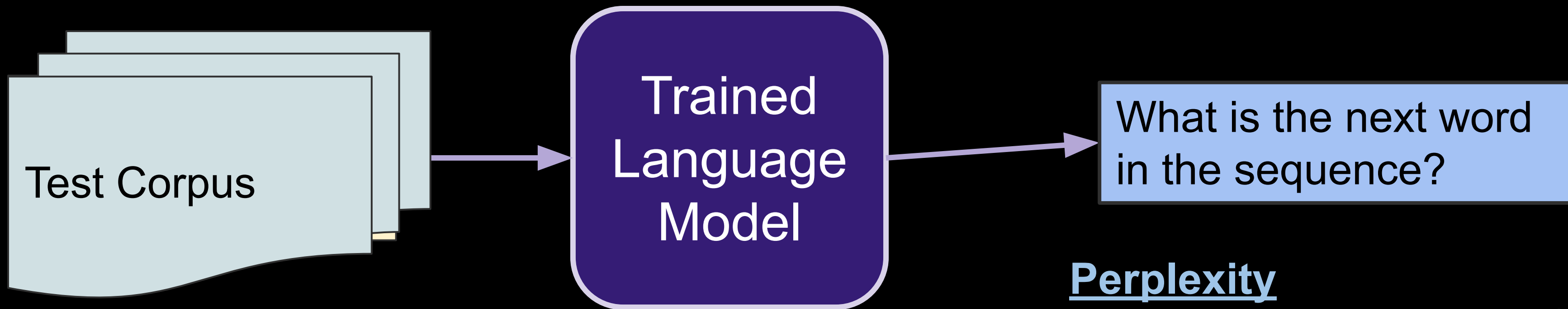


[Perplexity](#)

$$PP(W) = P(w_1 w_2 \ldots w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \ldots w_N)}}$$

Evaluation



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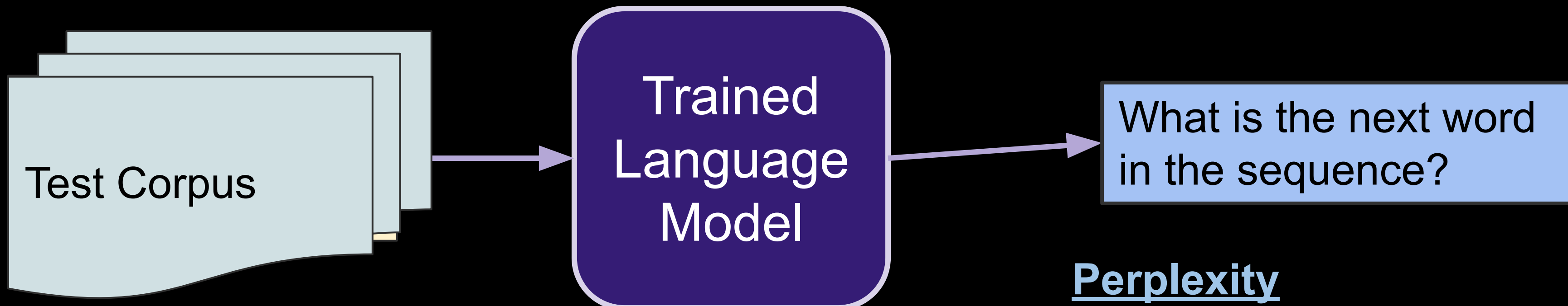

Perplexity

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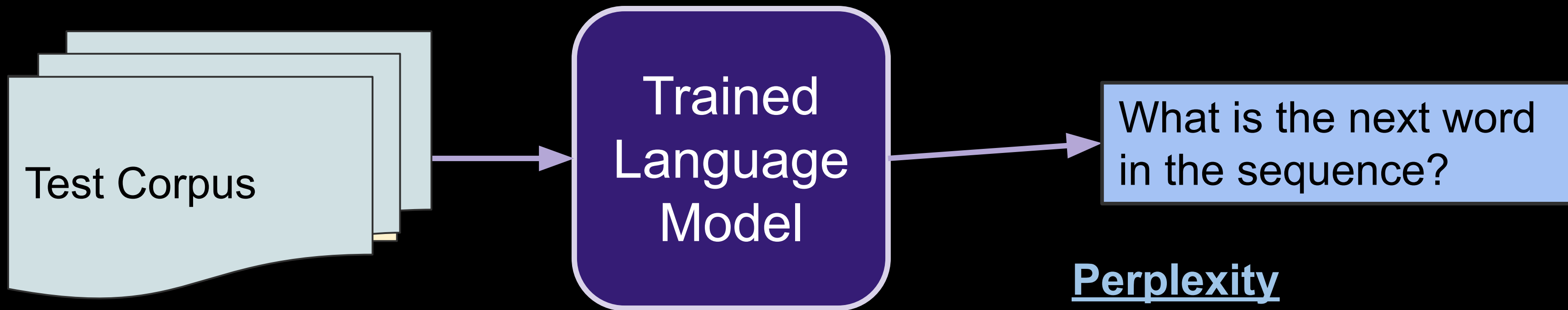
Apply Chain Rule: $\mathrm{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}}$

Evaluation



Perplexity

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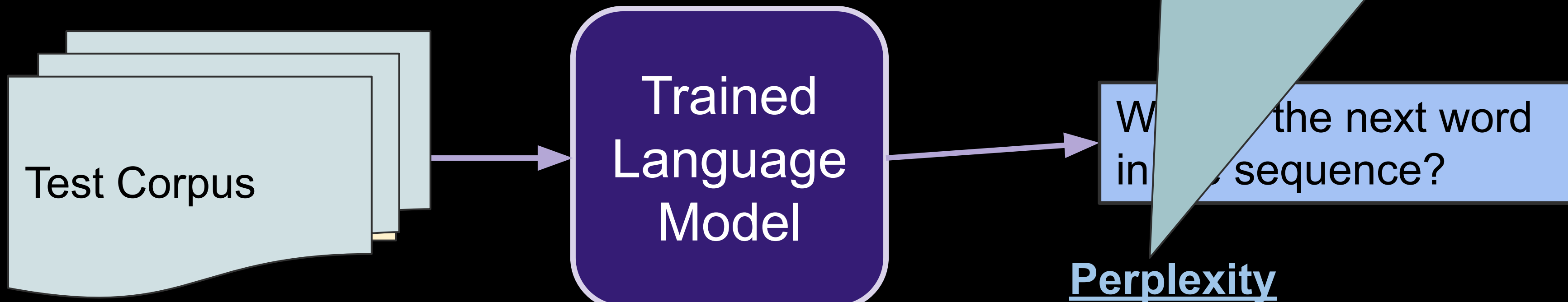
Apply Chain Rule: $$\text{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}}$$

Thus,
PP for Bigrams: $$\text{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_{i-1})}}$$

Evaluation

Reasoning:

1) Inverse of probability
   (i.e. minimize perplexity = maximize likelihood)
2) (weighted) average branching factor



$$PP(W) = P(w1w2w3...wN)^{1/N}$$

$$= \sqrt[N]{\frac{1}{P(w1w2w3...wN)}}$$

Apply Chain Rule: $$\text{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_1 \ldots w_{i-1})}}$$

Thus,
PP for Bigrams: $$\text{PP}(W) = \sqrt[N]{\prod_{i=1}^{N} \frac{1}{P(w_i|w_{i-1})}}$$

Practical Considerations:

- Use log probability to keep numbers reasonable and save computation. (uses addition rather than multiplication)
- Out-of-vocabulary (OOV)
  Choose minimum frequency and mark as <OOV>
- Sentence start and end: *<s> this is a sentence </s>*
  Advantage: models word probability at beginning or end.

Zeros and Smoothing

first word(X_{i-1}) \ second word (X_i)

$P(X_i \mid X_{i-1})$

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Example from (Jurafsky, 2017)

Zeros and Smoothing

first word \ second word

Bigram Counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Laplace (“Add one”) smoothing: add 1 to all counts

Zeros and Smoothing

Bigram Counts

first word \ second word

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Laplace (“Add one”) smoothing: add 1 to all counts

Unsmoothed probs

first word(X_{i-1}) \ second word (X_i)

$P(X_i \mid X_{i-1})$

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Example from (Jurafsky, 2017)

Smoothed

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

(vocabulary size)

first word(Xi-1) \ second word (Xi)

P(Xi | Xi-1)

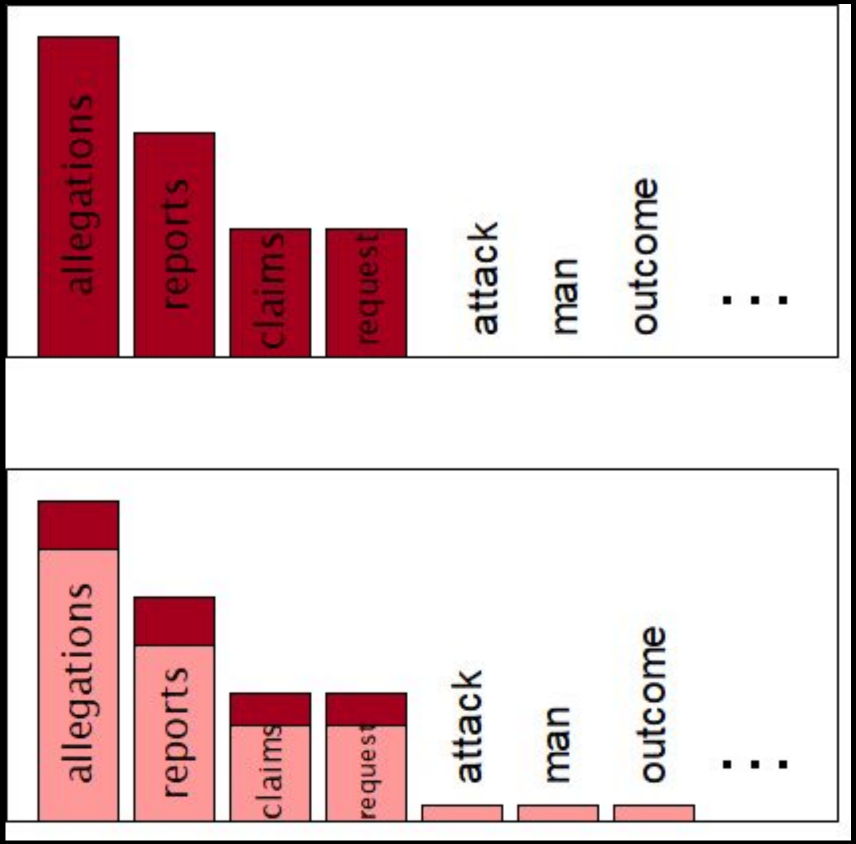
	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Example from (Jurafsky, 2017)

Why Smoothing? Generalizes

Original

With Smoothing



(Example from Jurafsky / Originally Dan Klein)

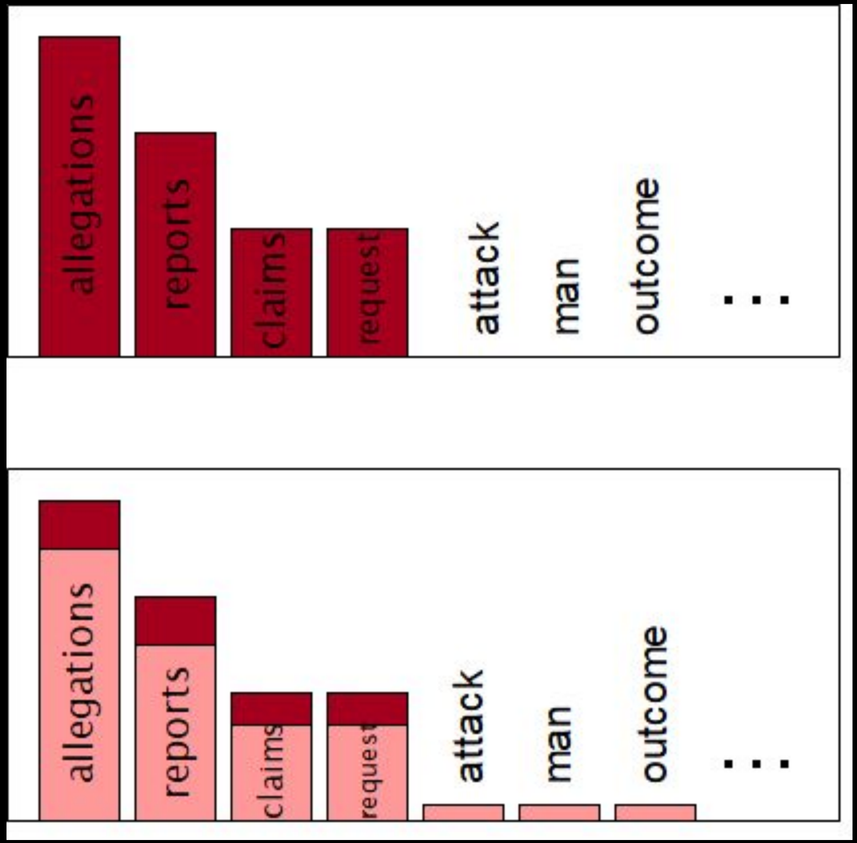
Why Smoothing? Generalizes

Add-one is blunt:
can lead to very large changes.

More Advanced:

Good-Turing Smoothing
Kneser-Nay Smoothing

These are outside scope for now.
We will eventually cover, even stronger, deep learning based models.



Why Smoothing?

What about Logistic Regression? Y = next word

P(Y|X) = P(Xn | Xn-1, Xn-2, Xn-3, ...)

Not a terrible option, but Xn-1 through Xn-k would be modeled as independent dimensions. Let's revisit later.

Why Smoothing?

What about Logistic Regression? Y = next word

P(Y|X) = P(Xn | Xn-1, Xn-2, Xn-3, ...)

Not a terrible option, but Xn-1 through Xn-k would be modeled as independent dimensions. Let's revisit later. Could use:

P(Xn | Xn-1, [Xn-1 Xn-2], [Xn-1 Xn-2 Xn-3], ...)

Example how to produce language generator

1. Count unigrams, bigrams, and trigrams
2. Train probabilities for unigram, bigram, and trigram models (over training)
   a. with smoothing
   b. without smoothing
3. Generate language: Given previous word or previous 2 words, take a random draw from what words are most likely to be next.

Trigram model when good evidence (high counts)

Backing off to bigram or even unigram

Limitation: Long distance dependencies

The horse which was raced past the barn tripped .

Language Modeling Summary

- Two versions of assigning probability to sequence of words
- Applications
- The Chain Rule, The Markov Assumption: $P(X_1, X_2, ..., X_n) = \prod_{i=1}^{n} P(X_i | X_{i-k}, X_{i-(k-1)}, ..., X_i)$
- Training a unigram, bigram, trigram model based on counts
- Evaluation: Perplexity
- Zeros, Low Counts, and Generalizability
- Add-one smoothing